Software System Design and Implementation

Functors, Applicative and Monads

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Data constructors revisited

```
data Point = Point Float Float
```

```
Point :: Float -> Float -> Point

Point 0.0 1.25 :: Point
```

Data constructors revisited

```
data Shape
   = Circle  Point Float
   | Path    [Point]
```

```
Circle :: Point   -> Float -> Shape
Path   :: [Point]          -> Shape
```

Data constructors revisited

```
data Tree a
   = Leaf
   | Node a (Tree a) (Tree a)
```

```
Leaf ::                                Tree a
Node :: a -> (Tree a) -> (Tree a) -> (Tree a)
```

Data constructors revisited

```
data Either a b
   = Left  a
   | Right b
```

```
Left  :: a -> Either a b
Right :: b -> Either a b
```

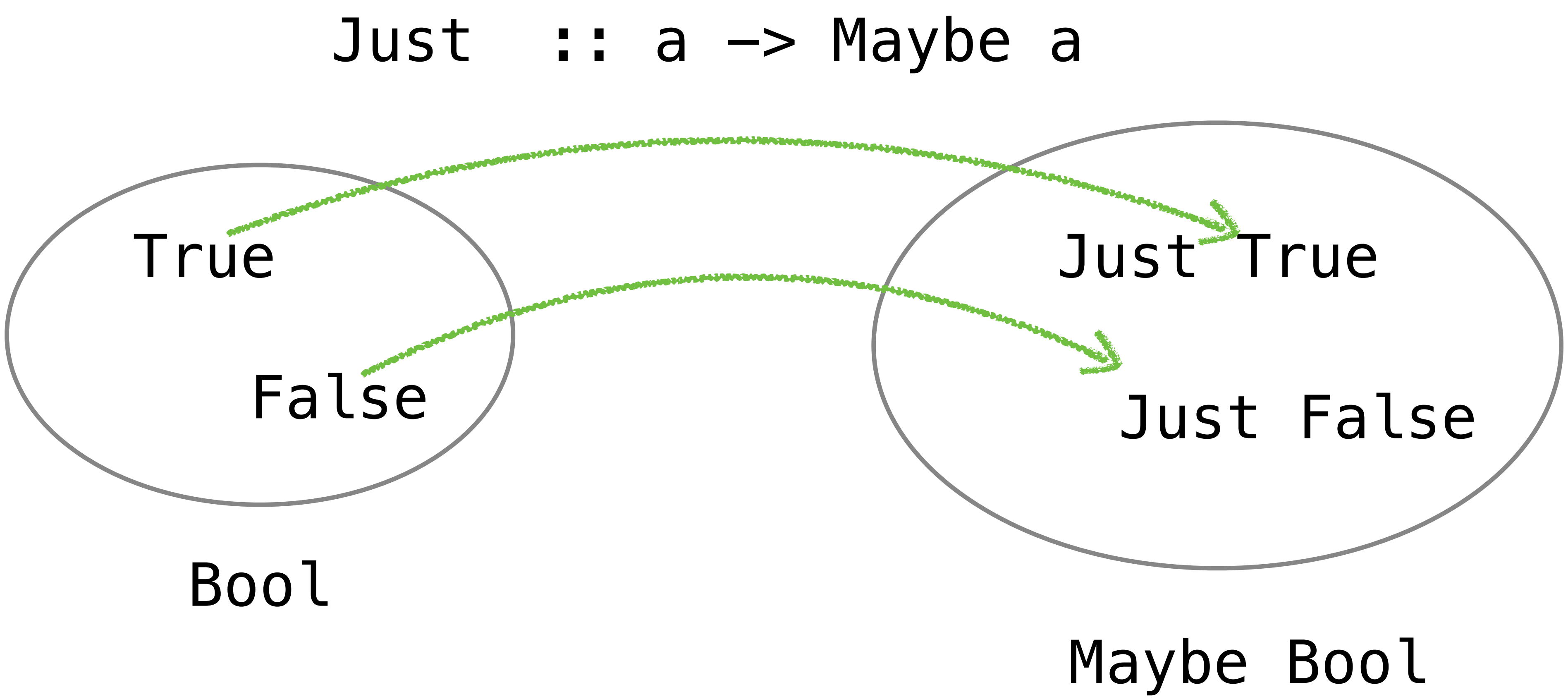
Data constructors revisited

```
data Maybe a
   = Nothing
   | Just a
```

```
Nothing ::      Maybe a
Just    :: a -> Maybe a
```

Type Constructors

- **Data constructors** map values to values:

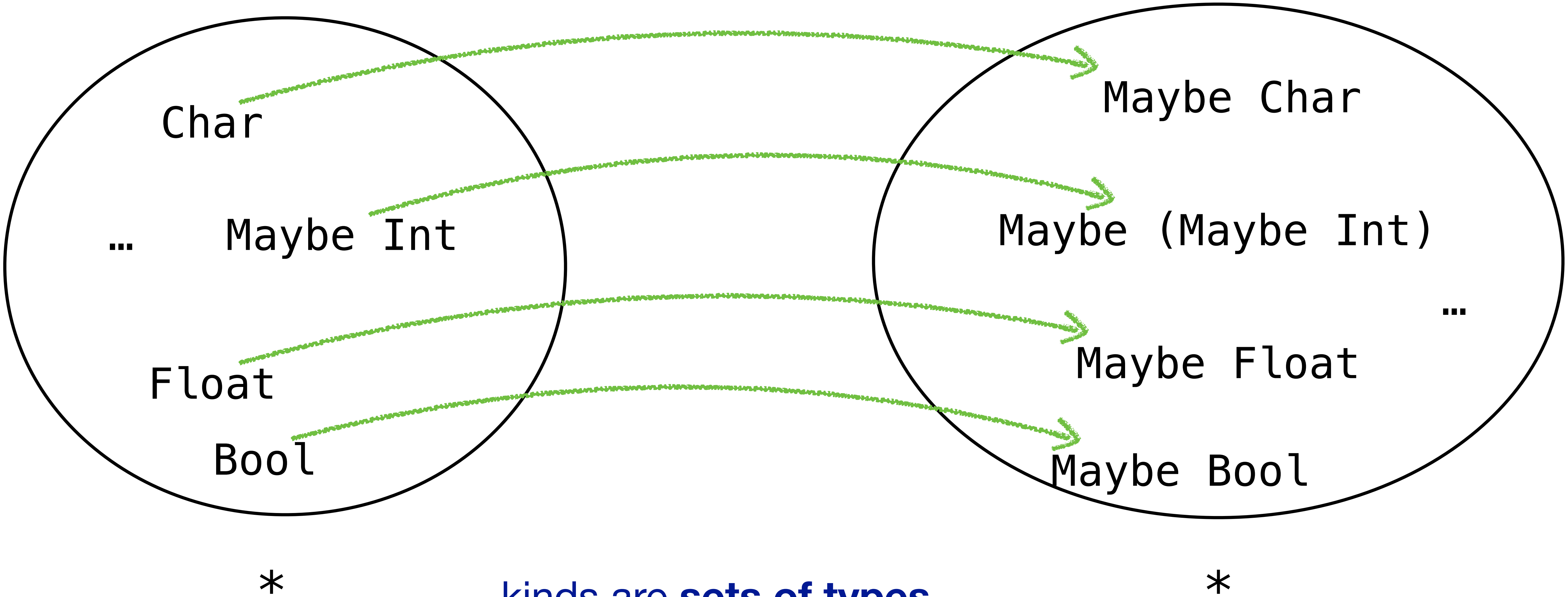


types a **sets of values**

Type Constructors

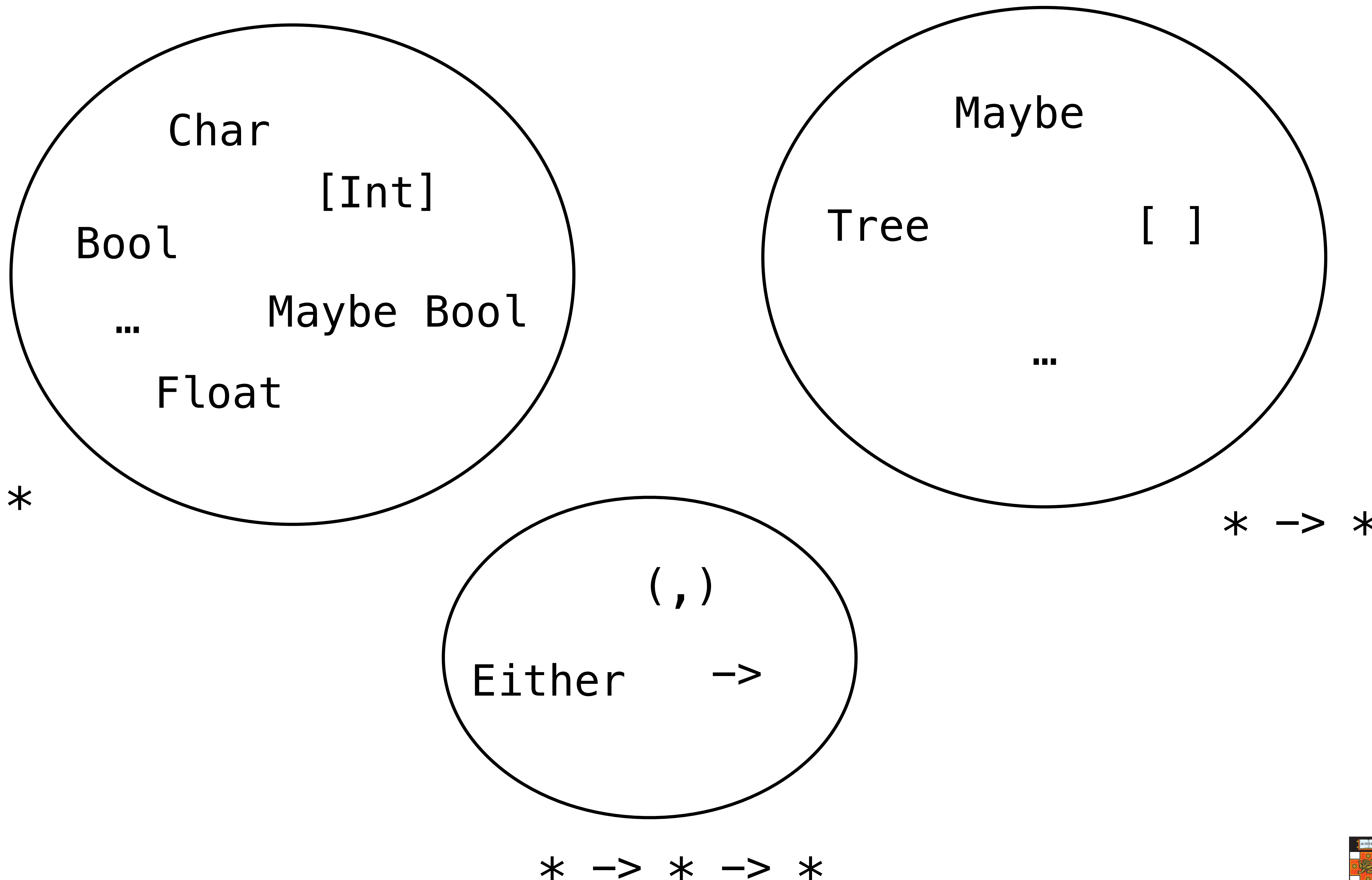
- **Type constructor** map types to type:

`Maybe :: * -> *`



kinds are **sets of types**

Kinds



Generalising map

- map on lists:

```
map :: (a -> b) -> [a] -> [b]
map f [] = []
map f (x : xs)  = f x : map f xs
```

- map for other unary type constructors:

```
treeMap :: (a -> b) -> Tree a -> Tree b
treeMap f Leaf = Leaf
treeMap f (Node x leftSubtree rightSubtree)
  = Node f x (treeMap f leftSubtree)
             (treeMap f rightSubtree)
```

Generalising map

- map on the Maybe type:

```
maybeMap :: (a -> b) -> Maybe a -> Maybe b
maybeMap f Nothing  = Nothing
maybeMap f (Just x) = Just (f x)
```

```
fmap :: (a -> b) ->  f a -> f b
```

Functors

- We have seen how type classes can be used to group types according to the operations supported on their values:

a :: *

```
class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

instance Eq Bool where
  (==) True   True  = True
  (==) False  False = True
  (==) _      _     = False
  (/=) b1     b2    = not (b1 == b2)
```

Functors

- We can also use type classes to group type constructors:

f :: * -> *

```
class Functor f where
   fmap :: (a -> b) -> f a -> f b


instance Functor Tree where
   fmap f Leaf   =  Leaf
   fmap f (Node a t1 t2)
      = Node (f a) (fmap f t1) (fmap f t2)
```

What properties should map have?

- Should leave the structure intact:

```
      fmap id xs  == xs

fmap (f . g) xs  == ((fmap f) . (fmap g)) xs
```

- These properties are not enforced by the compiler
  - it's the programmers responsibility to ensure
  - these are quickcheckable properties, but proofs are often straight forward
  - these abstractions are very useful to understand code

Applicative

- Applicative are functors with two additional operations:

```
class Functor f => Applicative f where

   pure  :: a -> f a
  (<*>)  :: f (a -> b) -> f a -> f b
```

Applicative

- Properties

```
              pure id <*> v == v

pure (.) <*> u <*> v <*> w == u <*> (v <*> w)

          pure f <*> pure x == pure (f x)

             u <*> pure y == pure ($ y) <*> u
```

Monads

- Monads

```
class Applicative m => Monad m where

  (>>=)  :: m a -> (a -> m b) -> m b
  return :: a -> m a
```

Monads

- Properties

```
     return a >>= k  ==  k a

       m >>= return  ==  m

m >>= (\x -> k x >>= h)  ==  (m >>= k) >>= h
```

Monads

- Do-notation:

```
incMaybe :: Num a => Maybe a -> Maybe a
incMaybe (Just x) = Just (x +1)
incMaybe _        = Nothing


incM mx
  = mx >>=  \x ->
    return (x + 1)

addM mx my
  = mx >>= \x ->
    my >>= \y ->
    return (x + y)

addM mx my = do
  x <- mx
  y <- my
  return (x + y)
```